Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name: Topology

Subject Code: 4SC06TOC1/4SC06TOP1 **Branch: B.Sc. (Mathematics)**

Semester: 6 **Date: 25/04/2019** **Time: 10:30 To 01:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Define: Subspace Topology. **1**

b) Let X be any non-empty set, write co-finite topology on X. **1**

c) Define:Open set. **1**

d) Define: Exterior of set. **1**

e) Define: Boundary point. **1**

f) Define: T_1 space. **1**

g) Define: Separated sets. **1**

h) Let (X,τ) be T_1 topological space then $\{x\}$ is closed $\forall x \in X$. True or False. **1**

i) Prove that $(A \cap B)^{\circ} = A^{\circ} \cap B^{\circ}$. **2**

j) Let $f: R \to R$ with $f(x) = x$ then check f is continuous or not. Justify your answer. **2**

k) Let X be a topological space and $A \subset X$ then prove that A is closed if and only if $A' \subset A$. **2**

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Let X be non-empty set. Let $\tau_1$and τ_2 be two topologies on X. Let $\tau = \tau_1 \cap \tau_2$. Then prove that τ is topology on X. Is $\tau_1 \cup \tau_2$ topology on X? Justify your answer. **6**

b) Let $X = R$. **5**
Let $\tau = \{U \subset X \mid \text{for x} \in \text{U there is } \epsilon > 0 \; such\ that \; (\text{x} - \epsilon, \text{x} + \epsilon) \subset \text{U} \}$then prove that τ is topology on R..

c) Compare lower limit topology and usual topology on R. **3**

Q-3 **Attempt all questions** **(14)**

a) Let $X = \{a,b,c,d\}$ and $\tau = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}\}$ **6**
(i) Find all open sets
(ii) Find all closed sets
(iii) If $A = \{a,d\}$, find $A^{\circ}, \bar{A}$.
(iv) If $B = \{b,c,d\}$ find $B^{\circ}, \bar{B}$

b) Let Y be a subspace of a topological space X then prove that a set A is closed in Y if and only if $A = Y \cap C$ where C is closed in X. **4**

	c)	Let (X, τ) be a topological space &A, B be two subsets of X then prove that, (i) If $A \subset B$ then $A' \subset B'$. (ii) $(A \cap B)' \subseteq A' \cap B'$. (iii) $A' \cup B' = (A \cup B)'$.	**4**
Q-4		**Attempt all questions**	**(14)**
	a)	Let X and Y be a topological space and $f: X \rightarrow Y$, then prove that following are equivalent (i) f is continuous. (ii) For every subset A of X then $f(\bar{A}) \subset \overline{f(A)}$. (iii) For every close set B in Y then $f^{-1}(B)$ is closed in X.	**6**
	b)	Let X be a topological space and A be a subset of X. Then prove that $\bar{A} = A \cup A'$.	**6**
	c)	Let (X, τ) be a topological space &A, B be two subsets of X then prove that $ext\ (A \cup B) = ext\ A \cap ext\ B$.	**2**
Q-5		**Attempt all questions**	**(14)**
	a)	Prove that homeomorphism is an equivalence relation in the collection of topological spaces.	**6**
	b)	Let (X, τ) be disconnected topological space and τ' is finer than τ. Prove that (X, τ') is disconnected.	**4**
	c)	Let (X, τ) be a topological space &A, B be two open subsets of X then prove that A & B are separated if and only if $A \cap B = \phi$.	**4**
Q-6		**Attempt all questions**	**(14)**
	a)	Let X be a topological space. Then X is disconnected if and only if there exists a non-empty subset of X which is both open and closed in X.	**6**
	b)	Prove that every subspace of T_1 space is T_1space.	**5**
	c)	Prove that indiscrete topology is not T_0 space.	**3**
Q-7		**Attempt all questions**	**(14)**
	a)	State and prove Heine Borel theorem.	**10**
	b)	Let X be co countable topological space then X is compact if and only if X is finite.	**4**
Q-8		**Attempt all questions**	**(14)**
	a)	Prove that continuous image of connected space is connected.	**5**
	b)	Prove that continuous image of compact space is compact.	**5**
	c)	Prove that R with usual topology is T_1 space.	**4**